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**RESPONSE OF CONVECTIVELY CONTROLLED BURNING  
TO NONLINEAR DISTURBANCES**

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# RESPONSE OF CONVECTIVELY CONTROLLED BURNING TO NONLINEAR DISTURBANCES

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## ABSTRACT

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A process where energy release rate is proportional to the square root of the Reynolds number of a burning particle or surface is used to demonstrate some distinctive dynamic properties of a disturbed convectively controlled burning process. The instantaneous energy release rates caused by assumed periodic flow field disturbances are numerically evaluated and examined. Correlation coefficients which express the energy released in-phase with the pressure disturbance are evaluated and analytical solutions for these coefficients are derived.

The response of the process is shown to be highly sensitive to the harmonic distortion of the disturbance. For some disturbances the energy released in-phase with the pressure disturbance is an order of magnitude larger than that for linear (sinusoidal) disturbances. The results show that harmonic distortion increases the coupling between the burning process and the flow field. This amplifying effect of harmonic distortion is suppressed when a disturbance becomes steep-fronted; therefore, steepening can act to limit the equilibrium amplitudes in disturbed systems. Amplification is also suppressed when the velocity disturbance is phase shifted with respect to the pressure disturbance or when the process is exposed to a high steady velocity.

## INTRODUCTION

The energy release process in many reactive systems depends on the convective forces on a burning particle or surface. For such systems, both the density and velocity associated with nonsteady flow will affect the rate of energy release. The purpose of this paper is to examine and evaluate the dynamic coupling between a convectively controlled burning process and a nonsteady flow field where pressure, density and gas velocity vary periodically. Emphasis is placed on distorted (nonsinusoidal) flow field disturbances which include these with "shock-like" wave shapes.

For most reactive systems, linear (small sinusoidal perturbation) analyses provide much of the basic insight into the behavior of the disturbed system. Linear analyses are valuable because one can readily comprehend the physical behavior described by the analysis. However, the distortion and gas motion which accompany many disturbances are neglected in linear analyses. The practitioner in real systems should know and understand the significance of these deviations from linearity. The method of analysis employed in this study is directed toward the objective of expanding upon the insight which linear analyses provide by including the affects of distortion and gas motions in the evaluation of dynamic properties.

A process where energy release rate is simply proportional to the Reynolds number of a burning particle or surface to an exponential power is used to demonstrate some distinctive dynamic properties of the convectively controlled burning process. The instantaneous energy release caused by assumed flow field disturbances are numerically evaluated and examined. A correlation coefficient which expressed the energy released in-phase with the

pressure disturbance is evaluated and analytical solutions for the coefficient are derived. The effect of distortion and gas velocity on the coefficient is discussed with regard to the growth and decay of the disturbance.

The method of analysis and some of the results have been reported previously (1,2) in studies of combustion instability in liquid propellant rocket engines. The broader implications of these previous studies are reviewed without the application to a specific reaction system. Additional analytical solutions and evaluations are presented for steep-fronted and "shock-like" disturbances which are frequently encountered in a variety of reactive systems.

#### MODEL

The model used to evaluate the response properties of a convectively controlled burning process to flow field disturbances includes the following elements:

##### Energy Release Process

A process is assumed where energy release rate is given by:

$$w = K_1 (Re)^{1/2} = K_1 \left( \frac{\rho D U}{\mu} \right)^{1/2} \quad (1)$$

An energy release rate dependent on Reynolds number, as expressed by equation (1), characterizes the convective heat and mass transfer mechanisms which dominate many energy release processes. The gas viscosity,  $\mu$ , and the characteristic dimension,  $D$ , are assumed constant. The velocity parameter  $U$  is the magnitude of the relative velocity seen by the particle or surface. For this study a two-dimensional flow field which is steady in one dimension

and periodic in the other is assumed. For such conditions  $U$  is defined by:

$$U^2 = \Delta \bar{v}^2 + u^2 \quad (2)$$

Combining equation (1) and (2) with the relation  $\rho = \bar{\rho}(1 + \rho')$ , an energy release rate normalized by the rate without perturbations is given by:

$$W = \frac{w}{K_1 \left( \frac{\bar{\rho} \Delta \bar{v}}{\mu} \right)^{1/2}} = (1 + \rho')^{1/2} \left[ 1 + \left( \frac{u}{\Delta \bar{v}} \right)^2 \right]^{1/4} \quad (3)$$

#### Flow Field Disturbances

The pressure, density and velocity are assumed to be periodically varying in the flow field. Nonlinear disturbances in these variables are assumed to be generally described by series of the form:

$$\begin{aligned} p' &= \sum_{n=1}^{\infty} p_n \cos(n\omega t - \varphi_n) \\ \rho' &= \sum_{n=1}^{\infty} p_n \cos(n\omega t - \varphi_n) \\ u &= \sum_{n=1}^{\infty} u_n \cos(n\omega t - \varphi_n - \theta_n) \end{aligned} \quad (4)$$

The angle  $\varphi_n$  (harmonic phase angle) gives the phase relation between the harmonic components. As in linear analyses, pressure and density disturbances are assumed to be in-phase with each other. The angle  $\theta_n$  (velocity-pressure phase angle) expresses the phase relation between the velocity and pressure disturbances. For traveling waves, which will often be discussed,

the velocity and pressure are assumed in-phase as in linear analyses.

Equation (4) is simplified for some specific evaluations in order to reduce the number of variables. The assumptions that  $p_n = \gamma p_n$  and  $u_n = c \rho_n$  are made. These are approximations adopted from linear theory and adequately characterize the gas dynamics for the demonstration purposes of this study.

Specific evaluations are made for two types of harmonically distorted disturbances. In one type the distortion is limited to two harmonic components. In the other type multiharmonic distortion is considered where the harmonic coefficients are assumed ordered by  $p_n = p_1^n$  and the phase angles  $\phi_n$  and  $\theta_n$  are assumed constant. Such highly ordered harmonic series can describe the "spiked" pressure wave shapes observed in strong traveling acoustic modes (4) as well as "shock-like" shapes.

#### Response Properties

The response properties are expressed by correlating parameters which relate the perturbation in energy release rate to the imposed disturbances in the flow field. One of the more important correlating parameters is the component of the energy release rate perturbation which is in-phase with the pressure disturbance. According to Rayleigh's criterion for heat driven waves, this in-phase response gauges the ability of the process to amplify or attenuate the flow field disturbance.

For this analysis the component of the energy release rate perturbation in-phase with the pressure disturbance will be called the in-phase response factor,  $\mathcal{R}$ . It can be extracted from the energy release rate perturbation

and normalized by a correlating procedure (1) defined by:

$$\mathcal{R} = \frac{\int_0^{2\pi} W'P' \, d\omega t}{\int_0^{2\pi} (P')^2 \, d\omega t} \quad (5)$$

By linear analysis, it can be shown that energy is added to the flow field disturbance when  $\mathcal{R}$  is positive and energy lost when negative. Growth or decay rate of the disturbance depends on the magnitude of  $\mathcal{R}$ . In unstable rocket combustions, where energy is continually lost due to exhaust flow,  $\mathcal{R}$  must usually be greater than about unity for sustained oscillations. Whether such linear criterion apply directly to nonlinear disturbances has not been rigorously established. However, applications such as that made to Priem and Guentert's nonlinear studies in reference 1 show that criterion established by linear analyses acceptably predict nonlinear behavior.

#### Analytical Solutions

Approximate analytical solutions for the response can be derived by expanding the burning rate expression (eq. 3) in a Taylor series. Details of this method are given in reference 1. By neglecting harmonic content higher than second order in the Taylor series and cross products of harmonic coefficients higher than the fourth order the following general solution for  $\mathcal{R}$  is obtained for  $\phi_1 = 0$ :

$$\mathcal{R} = \frac{1}{2} \left( \frac{\rho_1 p_1 + \rho_2 p_2}{p_1^2 + p_2^2} \right) + \left\{ \begin{array}{l} p_1 u_1 u_2 \cos(\varphi_2 + \theta_2 - \theta_1) \\ + 1/2 p_2 u_1^2 \cos(2\theta_1 - \varphi_2) \\ + 1/2 (\rho_1 p_1 + \rho_2 p_2) u_1 u_2 \cos \theta_1 \cos \theta_2 \\ + 1/8 \rho_1 p_1 u_1^2 \cos 2\theta_1 \\ + 1/8 \rho_2 p_2 u_2^2 \cos 2\theta_2 \\ + \frac{1}{4 \Delta v^2 (p_1^2 + p_2^2)} \left[ 1 + 1/2 \Delta v^2 (u_1^2 + u_2^2) \right] \end{array} \right\} \quad (7)$$

The harmonic coefficients and phase angles are shown to interact in various combinations to affect the response.

For the more specific disturbance where  $p_n = r \rho_n$  and  $u_n = c \rho_n$ , the solution takes the form:

$$\mathcal{R} = \frac{1}{2r} + \frac{\frac{1}{4r} \left( \frac{c}{\Delta v} \right)^2 \left( \frac{p_1}{r} \right)^2}{1 + \left( \frac{p_2}{p_1} \right)^2} \left\{ \begin{array}{l} \left( \frac{r}{p_1} \right) \left( \frac{p_2}{p_1} \right) \left[ \cos(\varphi_2 + \theta_2 - \theta_1) + \frac{1}{2} \cos(\varphi_2 - 2\theta_1) \right] \\ + \frac{1}{2} \left( \frac{p_2}{p_1} \right)^2 \cos \theta_1 \cos \theta_2 + \frac{r}{8} \cos 2\theta_1 + \frac{1}{8} \left( \frac{p_2}{p_1} \right)^4 \cos 2\theta_2 \\ 1 + \frac{1}{2} \left( \frac{c}{\Delta v} \right)^2 \left( \frac{p_1}{r} \right)^2 \left[ 1 + \left( \frac{p_2}{p_1} \right)^2 \right] \end{array} \right\} \quad (8)$$

In this solution, the amount of second harmonic distortion is expressed as a ratio of second to first harmonic amplitudes,  $p_2/p_1$ . The first term in equation (8),  $1/2r$ , is the linear response - the response to small sinusoidal disturbances.

The solution for  $\mathcal{R}$  to second order in  $p_1$  for the multiharmonic disturbances where  $p_n = p_1^n$  and  $\varphi$  and  $\theta$  are constants is given by:

$$\mathcal{R} = \frac{1}{2r} + \frac{\frac{1}{4r} \left( \frac{c}{\Delta v} \right)^2 \left( \frac{p_1}{r} \right)^2 \left[ r \cos \varphi + \frac{r}{2} \cos(\varphi - 2\theta) + \frac{1}{8} \cos 2\theta \right]}{1 + p_1^2 + \frac{1}{2} \left( \frac{c}{\Delta v} \right)^2 \left( \frac{p_1}{r} \right)^2} \quad (9)$$



In all of the solutions, the response factor  $\mathcal{R}$  is a maximum when the phase angles  $\phi$  and  $\theta$  are equal to zero. For equation (9) this condition of zero angles characterizes "spiked", traveling type disturbances.

#### DISCUSSION OF RESPONSE PROPERTIES

In the first part of this discussion the effect of the distortion in a disturbance on the instantaneous energy release rate will be described to illustrate some distinctive properties of the convectively controlled burning process. The instantaneous energy release rates were evaluated numerically with the aid of a digital computer. The discussion of the response factor  $\mathcal{R}$ , which follows, is also supplemented with some numerical evaluations which help establish the precision of the approximate analytical solutions for  $\mathcal{R}$ .

#### Instantaneous Energy Release Rates

The effect of harmonic distortion on the instantaneous energy release rates is best illustrated by considering the multiharmonic disturbances where  $p_n = p_1^n$ . For these disturbances, harmonic distortion increases with pressure amplitude. Figures 1-3 shows the change in the pressure wave shape with amplitude and compares the numerical solution for the energy release rate with the pressure disturbance for several conditions.

Figure 1 illustrates a type of reference condition in which velocity disturbances are neglected. The energy release rates are equivalent to those for a process which is only density sensitive ( $w \sim \rho^{1/2}$ ). Without velocity oscillations, the energy release rate essentially follows the pressure disturbance but at a reduced amplitude. The peak energy release rate varies from about one-half the peak pressure for disturbances which are sinusoidal (fig. 1(a)) to much less than one-half for the "spiked" distur-

bances (fig. 1(d)). There appears to be a loss in response with an increase in the harmonic distortion for a process which is only density sensitive.

Figure 2 illustrates the importance of velocity disturbance for the same pressure disturbances considered in figure 1. The effects of velocity disturbances are very pronounced for sinusoidal type disturbances (figs. 2(a) and (b)). The energy release rate perturbation is much larger than the pressure disturbance but exhibits two peaks. Two peaks exists because the convectively controlled process is sensitive to the magnitude and not the direction of the velocity vector and the magnitude is high at both high and low pressure conditions. With regard to coupling between the flow field and the process, the two peaks appear to have counteracting affects, that is, energy is released first in-phase and then out-of-phase with the pressure.

The correspondence between energy release rate and the pressure disturbances in figure 2 progressively improves with an increase in pressure amplitudes. When the amplitude and distortion are high (fig. 2(d)) the "spiked" disturbances cause a very similar disturbance in release rate. The second peak in release rate seen at low amplitudes is suppressed but the peak which is in-phase with the pressure peak remains large. The energy release rate perturbation appears as an amplified pressure disturbance.

Figure 3 illustrates the dynamic behavior for disturbances which become steep-fronted and "shock-like" with an increase in pressure amplitude. The results are very similar to those for the symmetrical type "spiked" disturbances illustrated in figure 2. An increase in amplitude and harmonic distortion improves the correspondence between the energy release rate and pressure disturbance.

These effects of harmonic distortion on the dynamic behavior of convectively controlled burning can be discussed from a more quantitative standpoint by a comparison of the in-phase response factors which relate the energy release rates to the pressure disturbances.

#### Response Factor

The effect of an increase in the harmonic content of a disturbance on the response factor  $\mathcal{R}$  is the best illustrated by initially considering disturbances which contain only first and second harmonic components. The analytical solution given by equation (8) applies to such disturbances. Figure 4 shows the effect of second harmonic distortion on the wave shape and on the in-phase response factor  $\mathcal{R}$  for disturbances where the harmonic phase angles  $\phi$  and  $\theta$  are equal to zero. The response factor exhibits optimum properties as a function of both pressure amplitude ( $P'_{rms}$ , the root-mean-square amplitude of the disturbance) and second harmonic content ( $p_2/p_1$ , the ratio of second to first harmonic pressure amplitudes). Maximum response occurs with a second harmonic content of about 0.8 at an amplitude of 0.02. This maximum is about an order of magnitude larger than that for linear (sinusoidal) disturbances which is shown by the  $p_2/p_1 = 0$  curve in figure 4. The result implies that second harmonic distortion can have a very large effect on the growth of a disturbance.

The effect of increasing the harmonic content in disturbances with multiharmonic distortion is shown in figure 5. The analytical solution given by equation (9) applied to such disturbances. For figure 5, the harmonic and velocity-pressure phase angles are equal to zero. These disturbances are of the type considered in figure 2. Figure 5 shows that an in-

crease in pressure amplitude which increases the harmonic distortion causes an increase in the response factor  $\mathcal{R}$  for any constant value of relative Mach number,  $(\Delta v/C)$ . At low amplitudes, the response is the linear value of  $1/2\gamma$  and it asymptotically approaches a value of about 3 times this linear value at high amplitudes. The result implies that in some systems a disturbance above some finite amplitude may be needed before the response is sufficiently large to cause a disturbance to grow in amplitude. Rocket engine combustors exhibit such behavior.

Figure 5 also shows the effect of the steady velocity  $(\Delta v/C)$  on the response. The convectively controlled process is more sensitive to velocity oscillations when the steady velocity environment of a particle or surface is small. Figure 5 shows that the response factor increases with a decrease in the steady velocity. For any real process, however, there is a lower limit to this steady velocity where convective forces no longer control the rate of energy release and this analysis is no longer applicable.

Analytical and numerical solutions for the response factor can be compared in figures 4 and 5. Such comparisons have shown that the analytical solutions generally over-predict the response by about 10 percent but adequately express the functional dependence of the response factor on the distortion variables.

The harmonic phase angle  $\phi$  affects the response factor as shown in figure 6. As  $\phi$  increases, the disturbance changes from being symmetrical about the peak as shown in figure 2 and becomes steep-fronted like those shown in figure 3. Such wave steepening caused by an increase in  $\phi$  is shown in figure 5 to suppress the increase in  $\mathcal{R}$  with an increase in pres-

sure amplitude. At an angle of  $\pi/2$  the response remains near the linear value of  $1/2\gamma$  even at high pressure amplitudes. This result implies that wave steepening may be a mechanism whereby pressure disturbances reach limiting amplitudes in reactive systems. That is, if a disturbance steepens and becomes "shock-like" as it grows in amplitude the energy added in-phase with the disturbance would decrease and eventually reach a condition where no more growth is possible.

For all the examples discussed thus far, the velocity disturbances have been in-phase with the pressure disturbance ( $\theta = 0$ ). Figure 7 shows the effect on the response factor of varying this phase relationship. An increase in the velocity-pressure phase angle  $\theta$  is shown to suppress the in-phase response factor in a manner similar to that for the harmonic phase angle  $\phi$ , but the effect is less severe. At an angle of  $\pi/2$ , the response  $R$  remains above the linear value of  $1/2\gamma$  at high amplitudes. In reactive systems, the velocity-pressure phase relation usually deviates from in-phase properties when the disturbances are reflected off of walls or exhibit standing-mode properties of acoustic resonance. Such disturbances should grow less rapidly than traveling wave disturbances.

#### CONCLUDING REMARKS

This analysis has shown the response properties of a convectively controlled burning process to be highly sensitive to the harmonic distortion of a flow field disturbance. Harmonic distortion can amplify or attenuate the response in a manner which should significantly alter the growth or decay of a disturbance. Directing more attention toward these response

properties of convectively controlled burning could increase the understanding of the dynamic behavior of many reactive systems and provide new approaches to experimentally modify the behavior of a specific system.

## APPENDIX - SYMBOLS

C	speed of sound
D	characteristic dimension
n	harmonic order
P	pressure
P' <sub>rms</sub>	root-mean-square pressure amplitude $\left( \frac{1}{2} \sum_{n=1}^{\infty} p_n^2 \right)^{1/2}$
p <sub>n</sub>	harmonic coefficient for pressure disturbance of order n
Re	Reynolds number
R	in-phase response factor, equation (5)
t	time
U	magnitude of relative velocity vector
u	gas velocity
u <sub>n</sub>	harmonic coefficient for velocity disturbance of order n
$\Delta \bar{v}$	steady relative velocity of particle or surface with respect to the gas
W	dimensionless energy release rate
w	energy release rate per unit time and volume
$\gamma$	ratio of specific heats
$\theta_n$	harmonic phase angle for velocity-pressure relation
$\rho$	gas density
$\rho_n$	harmonic coefficient for density disturbance of order n

$\phi_n$  harmonic phase angle

$\mu$  gas viscosity

$\omega$  frequency

' primed quantities denote dimensionless perturbations,  $x' = x - \bar{x}/\bar{x}$

- barred quantities denote mean values

END



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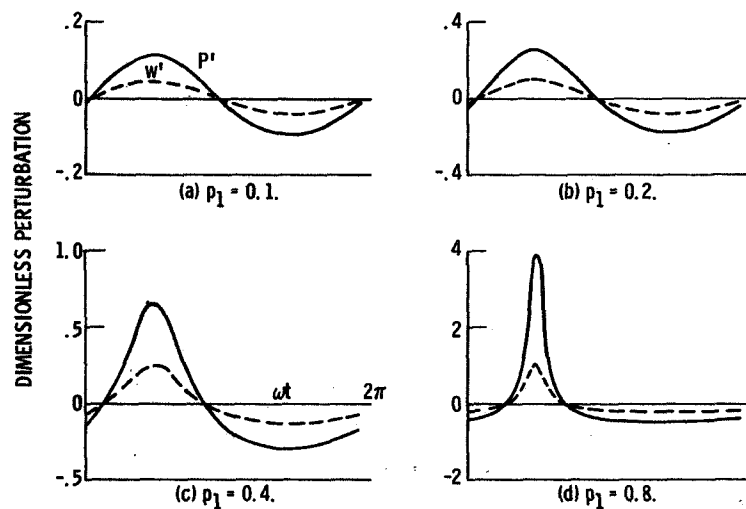


Figure 1. - Comparison of instantaneous energy release rates and pressure disturbances. Conditions:  $P' = \sum_{n=1}^{\infty} p_1^n \cos n\omega t$ ;  $u = 0$ .

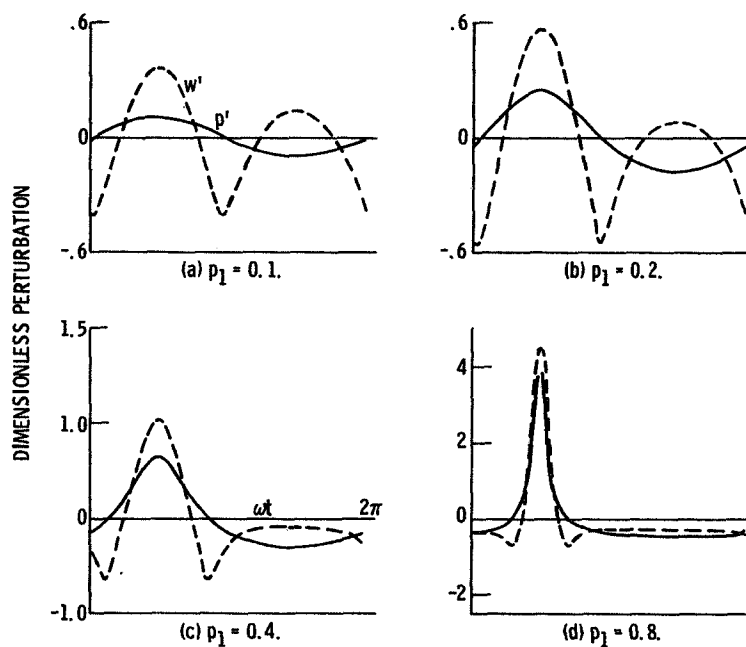


Figure 2. - Comparison of instantaneous energy release rates and pressure disturbances. Conditions:  $P' = \sum_{n=1}^{\infty} p_1^n \cos n\omega t$ ;  $u = \frac{c}{\gamma} \sum_{n=1}^{\infty} p_1^n \cos n\omega t$ .

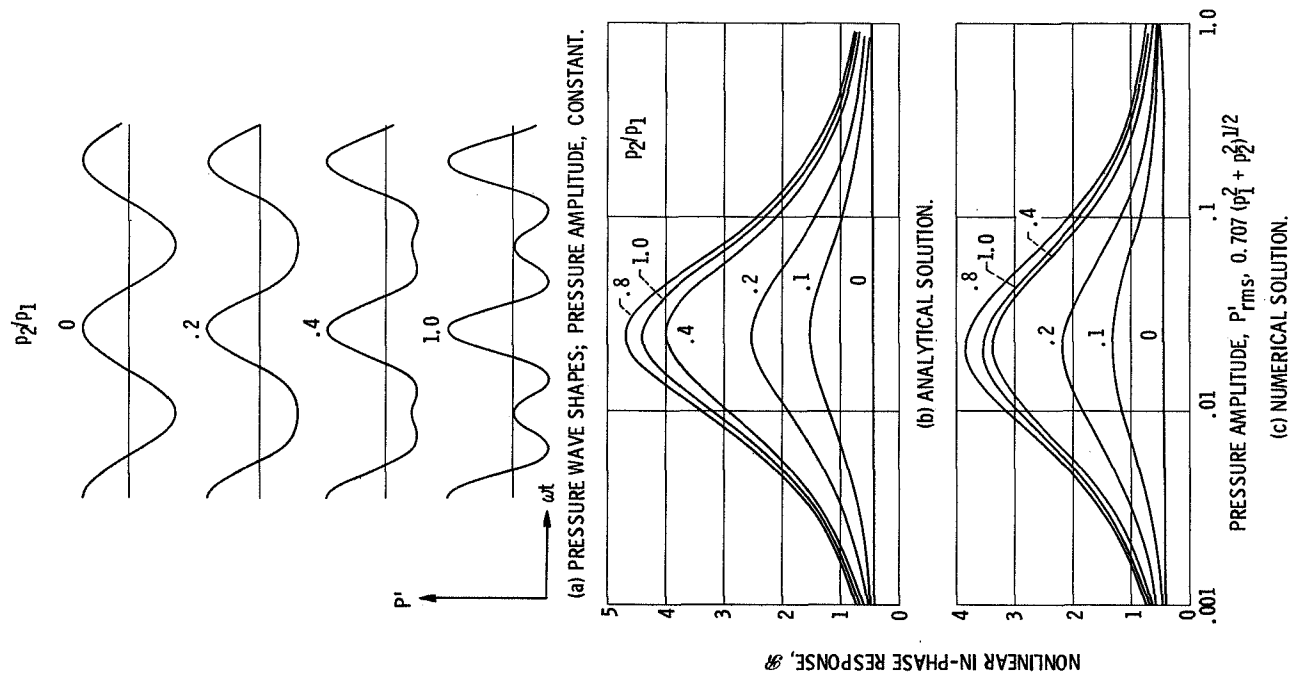


Figure 4. - Effect of second harmonic content on response and wave shape.  $\varphi_2 = 0$ ,  $\theta_1 = \theta_2 = 0$ ,  $\Delta v/c = 0.02$ ,  $\gamma = 1.2$ .

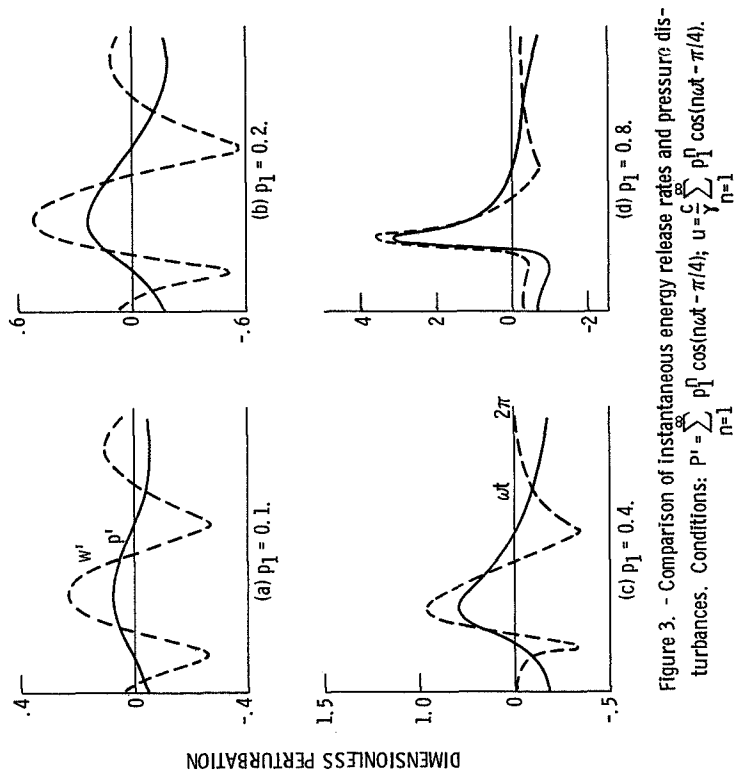
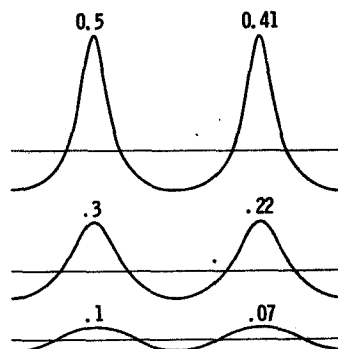
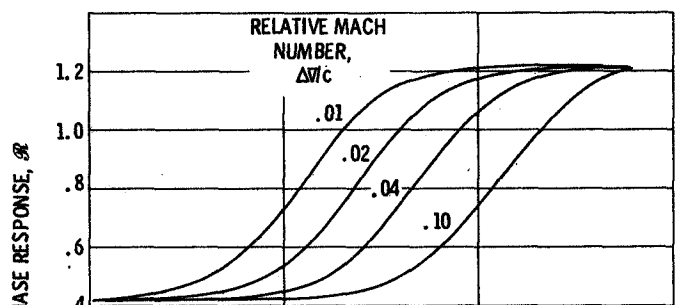


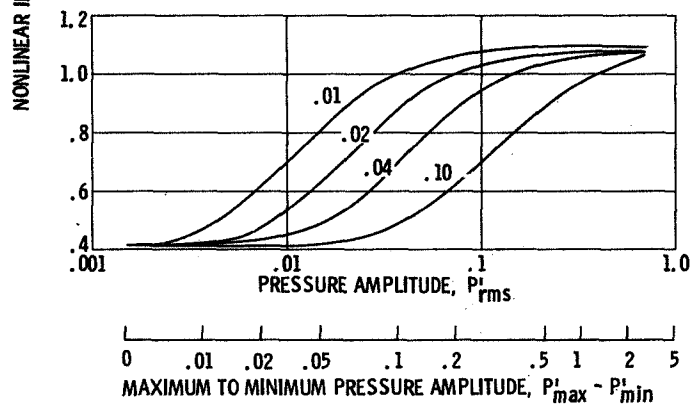
Figure 3. - Comparison of instantaneous energy release rates and pressure disturbances. Conditions:  $p' = \sum_{n=1}^{\infty} p_1^n \cos(n\omega t - \pi/4)$ ,  $u = \sum_{n=1}^{\infty} p_1^n \cos(n\omega t - \pi/4)$ .



(a) PRESSURE WAVE SHAPES.



(b) ANALYTICAL SOLUTION.



(c) NUMERICAL SOLUTION.

Figure 5. - Response and wave shapes for multiharmonic disturbances.  $\phi = 0$ ,  $\theta = 0$ ,  $\gamma = 1.2$ .

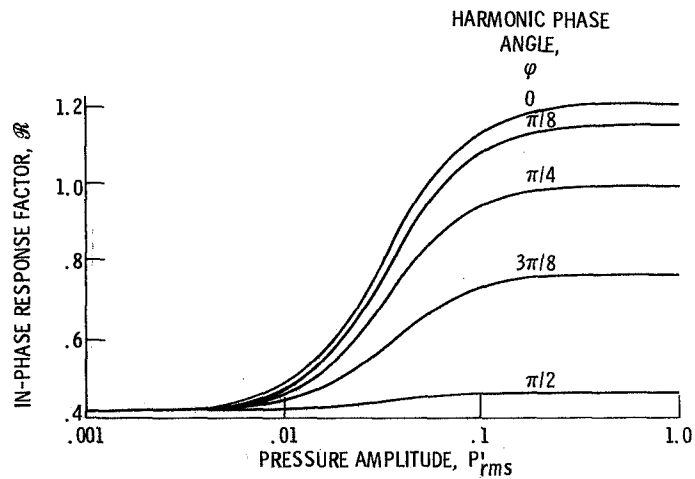


Figure 6. - Effect of harmonic phase angle on the response to multiharmonic disturbances equation (9).  $\theta = 0$ ,  $\Delta v/c = 0.02$ ,  $\gamma = 1.2$ .

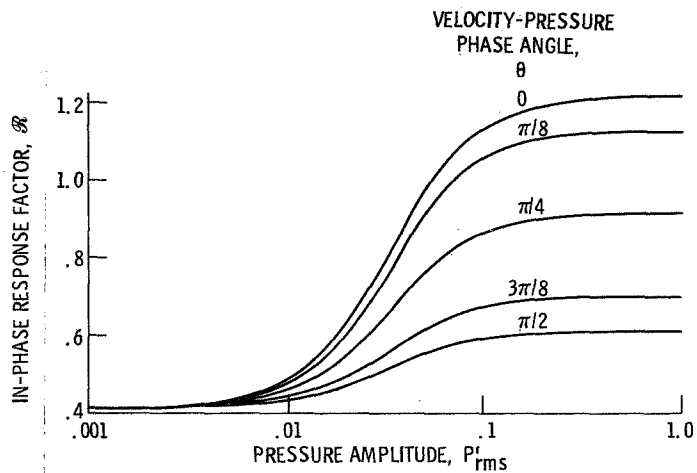


Figure 7. - Effect of velocity-pressure phase angle on the response to multiharmonic disturbances equation (9).  $\phi = 0$ ,  $\Delta v/c = 0.02$ ,  $\gamma = 1.2$ .